Module 4: Tracking and Resolution in Radar

This module focuses on the crucial aspects of accurately following detected targets and enhancing a radar system's ability to distinguish between closely spaced objects. We will explore the fundamental concepts of target tracking, delve into methods for precise angular measurement, and review advanced algorithms that form the backbone of modern radar surveillance and guidance.

4.1 Tracking Radar Principles

Once a target is detected by a radar, merely knowing its instantaneous position is often insufficient. For many applications (e.g., air traffic control, missile guidance, weather prediction), it is essential to determine the target's trajectory, predict its future position, and estimate its velocity and acceleration. This process is known as target tracking.

4.1.1 Introduction to Target Tracking Concepts

Target tracking in radar involves continuously estimating the kinematic state (position, velocity, acceleration) of one or more targets over time using a sequence of noisy radar measurements. The primary goal is to provide a smooth, accurate, and stable estimate of the target's path, even in the presence of measurement errors, target maneuvers, and missed detections.

Key concepts in target tracking include:

- **State Vector: The target's kinematic state at any given time is represented by a state vector. For a two-dimensional scenario, a simple state vector might include:**
    - **Position in X (x)**
    - **Position in Y (y)**
    - **Velocity in X ($\dot{x}$)**
    - **Velocity in Y ($\dot{y}$)**

    A more comprehensive state vector might also include acceleration components or other parameters.
- **Measurement Vector: This consists of the raw data provided by the radar for each detection, typically:**
    - **Range (R)**
    - **Azimuth Angle (θ)**
    - **Elevation Angle (ϕ) (for 3D radars)**
    - **Doppler Velocity (vd) (if available)**
- **Prediction: Based on the current estimated state of a track, the tracking algorithm predicts the target's future position and velocity for the time when the next measurement is expected. This prediction uses a target motion model (e.g., constant velocity, constant acceleration).**

- **Association:** When new radar measurements arrive, the tracking system must determine which measurements correspond to existing tracks (if any) and which might represent new targets or clutter. This is often the most challenging aspect in multi-target environments.
- **Update (Correction):** If a measurement is successfully associated with a track, the tracking algorithm uses this new measurement to refine and update the estimated state of the track, reducing the uncertainty in the estimate.
- **Track Initiation:** The process of establishing a new track when a series of consecutive detections appear to belong to a new target.
- **Track Termination:** The process of discontinuing a track when a target is no longer being detected for an extended period, or if its behavior suggests it is no longer of interest.

4.1.2 Various Tracking Methods (e.g., Track-While-Scan)

Radar tracking methods can be broadly categorized based on how the radar antenna is used in relation to the tracking process:

- **Single-Target Track (STT) Radar / Conical Scan:**
    - **Principle:** In this method, the radar antenna actively follows a single target. The antenna beam is continuously pointed at the target. Early STT radars used techniques like conical scan, where the beam rotated slightly around the target's anticipated direction. Any deviation of the target from the center of this rotation would generate an error signal that was used to steer the antenna to recenter on the target.
    - **Advantages:** Provides very accurate angular measurements and good signal-to-noise ratio for the tracked target due to continuous illumination.
    - **Disadvantages:** Can only track one target at a time. The radar cannot simultaneously search for new targets or track multiple existing ones. This limits its utility in multi-target environments. It's also susceptible to certain types of jamming.
    - **Application:** Historically used in fire control radars for engaging a single threat.
- **Track-While-Scan (TWS) Radar:**
    - **Principle:** TWS radar systems are designed to simultaneously search for new targets and track multiple existing targets while continuously scanning a designated volume of space. Instead of dedicating the beam to a single target, the radar periodically illuminates the entire surveillance volume. When new detections occur, they are correlated with existing tracks. If a detection falls within the predicted gate of an existing track, it is used to update

that track's state. If a detection does not correlate with an existing track, it may be used to initiate a new track.

  - Advantages: Enables true multi-target tracking capability. The radar maintains situational awareness across its entire coverage area. More efficient use of radar time and resources for surveillance and tracking.
  - Disadvantages: Angular accuracy per scan is typically less than dedicated STT, as the target is only illuminated periodically. Requires sophisticated data processing to associate detections with tracks and manage track files. The update rate for individual tracks is limited by the scan rate.
  - Application: Widely used in air traffic control, air surveillance, naval combat systems, and ground-based air defense radars. Modern phased array radars are particularly well-suited for TWS due to their agile beam steering capabilities.

In-depth Explanation:

The core of TWS is the track filter (e.g., Kalman Filter, discussed later). This filter takes the noisy measurements from each scan and processes them to produce a smooth estimate of the target's state. It also maintains a "track gate" around the predicted position of each target. New detections falling within this gate are considered candidates for association with that track. If no detection is found within the gate for several consecutive scans, the track may be put into a "coast" mode (predicting without updates) and eventually terminated. Handling maneuvers and false alarms efficiently are critical challenges in TWS.

4.2 Angular Resolution

Angular resolution is a crucial aspect of a radar system's ability to discriminate between targets that are close to each other in terms of their direction (azimuth or elevation). Just as range resolution determines how well a radar can separate targets along the same line of sight, angular resolution defines its capability to distinguish targets that are at the same range but at slightly different angles from the radar.

4.2.1 Definition and Factors Affecting Angular Resolution (Antenna Beamwidth)

Angular Resolution is defined as the minimum angular separation between two targets at the same range that the radar can distinguish as two separate entities rather than a single, larger target. This capability is directly related to the radar's antenna beamwidth.

The radar's antenna transmits and receives energy within a specific angular spread, known as its beamwidth. The smaller the beamwidth, the more precisely the radar can pinpoint the angular location of a target, and the better its angular resolution.

Typically, angular resolution is approximated by the half-power beamwidth (θ_{HP} or ϕ_{HP}), which is the angular extent between the points where the antenna's radiation pattern falls to half of its maximum power (or 3 dB down from the peak). If two targets are separated by an angle greater than the beamwidth, they are generally considered resolvable. If they fall within the same beamwidth, they will appear as a single, merged target unless special processing techniques are employed.

Factors Affecting Angular Resolution (Antenna Beamwidth):

The beamwidth of a radar antenna is primarily determined by two factors:

1. **Antenna Aperture Size (D): This refers to the physical dimensions of the antenna in the plane perpendicular to the direction of propagation (e.g., the diameter of a parabolic dish, or the length of a linear array).**
   - **Inverse Relationship: Angular resolution is inversely proportional to the antenna's aperture size. A larger antenna aperture produces a narrower beamwidth and thus better angular resolution. This is a fundamental principle of antenna theory.**
2. **Wavelength (λ): This is the wavelength of the transmitted radar signal.**
   - **Direct Relationship: Angular resolution is directly proportional to the wavelength. A shorter wavelength (higher frequency) results in a narrower beamwidth for a given antenna size, leading to better angular resolution.**

The approximate formula for the half-power beamwidth (θ_{HP}) in radians for a conventional antenna (like a uniformly illuminated rectangular or circular aperture) is:

$$\theta_{HP} \approx k \frac{\lambda}{D}$$

where:

- **θ_{HP} is the half-power beamwidth in radians.**
- **λ is the wavelength of the radar signal.**
- **D is the effective aperture dimension of the antenna in the plane of measurement (e.g., horizontal dimension for azimuth beamwidth, vertical for elevation beamwidth).**
- **k is a constant that depends on the antenna's aperture illumination (e.g., $k \approx 0.886$ for a uniformly illuminated rectangular aperture, $k \approx 1.02$ for a**

uniformly illuminated circular aperture). For a general approximation, $k \approx 1$ is often used.

To convert to degrees: $\theta_{HP}\ (\text{degrees}) = \frac{180}{\pi}\theta_{HP}\ (\text{radians})$.

In-depth Explanation:

The relationship between aperture size and beamwidth is a consequence of diffraction. A larger aperture provides more "sampling points" for the incoming wavefront, allowing for finer discrimination of arrival angles. Higher frequencies (shorter wavelengths) mean more cycles of the wave fit within the same physical aperture, providing more detailed phase information across the aperture, which also helps in angular discrimination. This is why high-resolution imaging radars often operate at very high frequencies (e.g., millimeter-wave bands) with large antennas.

Numerical Example:

A C-band air surveillance radar operates at a frequency of 5.6 GHz. It uses a parabolic dish antenna with a horizontal diameter of 5 meters.

Calculate the approximate horizontal (azimuth) half-power beamwidth in degrees.

Given:

$f = 5.6\ \text{GHz} = 5.6\times10^{9}\ \text{Hz}$

$D = 5\ \text{m}$

$c = 3\times10^{8}\ \text{m/s}$

Step 1: Calculate the wavelength (λ)

$\lambda = c/f = (3\times10^{8}\ \text{m/s})/(5.6\times10^{9}\ \text{Hz}) \approx 0.05357\ \text{m}$

Step 2: Calculate the beamwidth in radians (using $k \approx 1$ for approximation)

$\theta_{HP}\ (\text{radians}) \approx \lambda/D = 0.05357\ \text{m}/5\ \text{m} = 0.010714\ \text{radians}$

Step 3: Convert beamwidth to degrees

$\theta_{HP}\ (\text{degrees}) = 0.010714\ \text{radians} \times \frac{180}{\pi} \approx 0.614\ \text{degrees}$

The radar has an approximate horizontal half-power beamwidth of 0.614 degrees. This indicates good angular resolution, meaning it can distinguish targets separated by more than this angle at the same range.

4.3 Monopulse Technique

The Monopulse technique is an advanced method used by radar systems to achieve highly accurate angular measurements (azimuth and elevation) of a target within a single radar pulse. Unlike sequential lobing or conical scan methods that require multiple pulses to derive angle error, monopulse systems extract angle information from a single pulse, making them highly robust to target fluctuations and electronic countermeasures.

4.3.1 Principles of Amplitude and Phase Monopulse

The core principle of monopulse radar is to simultaneously illuminate the target with two (or more) slightly displaced antenna beams and compare the signals received from these beams to derive angle error. This comparison can be based on either amplitude differences or phase differences.

Amplitude Monopulse:

- **Principle: Uses multiple feed horns (or elements) in the antenna's focal plane to create overlapping antenna beams that are squinted (offset) slightly from the antenna's boresight (main axis). For example, to measure azimuth, two beams are formed: one squinted slightly left (L) and one squinted slightly right (R).**
- **Sum (Σ) and Difference (Δ) Patterns:**
  - **The Sum pattern (Σ) is formed by summing the signals from the two squinted beams (S_L+S_R). This pattern is broad and centered on the antenna's boresight, providing the maximum signal strength when the target is on boresight. It is used for detection and range measurement.**
  - **The Difference pattern (Δ) is formed by subtracting the signals from the two squinted beams (S_L-S_R). This pattern has a null (zero point) exactly on boresight and its amplitude increases rapidly as the target moves off boresight. The sign of the difference signal indicates the direction of the error (e.g., positive for left, negative for right).**
- **Angle Error Derivation: The angle error (ϵ) is proportional to the ratio of the difference signal magnitude to the sum signal magnitude:**
  $\text{Angle Error} \propto \Sigma \text{Re}(\Delta)$ (for amplitude monopulse, usually normalized)
  When a target is exactly on boresight, the difference signal is zero, indicating no angle error. Any deviation produces a non-zero difference signal whose magnitude is related to the angular displacement and whose phase (relative to the sum signal) indicates the direction.
- **Implementation: Typically involves a hybrid junction (microwave circuit) that takes the signals from two feed horns and produces sum and difference outputs.**

Phase Monopulse:

- **Principle: Uses two (or more) antenna elements spaced apart by a certain distance. The beams generated by these elements are parallel but physically displaced. When a target is off boresight, the signal arrives at the two elements with a phase difference, rather than an amplitude difference.**
- **Sum (Σ) and Difference (Δ) Patterns: Similar sum and difference patterns are formed, but here the difference signal is obtained by phase comparison rather than amplitude comparison.**
- **Angle Error Derivation: The angle error is derived from the phase difference between the signals received by the two elements. If the signals from elements 1 and 2 are V1 and V2, the phase difference $\Delta\phi = \text{phase}(V1) - \text{phase}(V2)$. The angle off boresight is approximately proportional to $\Delta\phi$.**
- **Implementation: Often used in phased array radars, where phase shifters can create the required phase gradients to form squinted beams or measure phase differences across the array.**

Two-Axis Monopulse:

To obtain both azimuth and elevation angle measurements, a monopulse radar typically employs four feeds/elements (e.g., in a square configuration). These four signals are combined to produce:

- **One Sum (Σ) channel (for range and overall detection).**
- **One Azimuth Difference (Δaz) channel.**
- **One Elevation Difference (Δel) channel.**
  The angle errors in both azimuth and elevation are then simultaneously derived from these difference signals relative to the sum signal.

4.3.2 Sum and Difference Patterns and Their Application in Precise Angle Estimation

The unique characteristics of the sum and difference patterns are fundamental to monopulse operation:

- **Sum (Σ) Pattern:**
  - **Shape: Typically a conventional, broad, single-lobed antenna pattern centered on boresight.**
  - **Purpose: Provides the primary detection capability. Its amplitude is maximum when the target is on boresight, ensuring maximum signal-to-noise ratio for detection and range measurement. It serves as a reference for normalizing the difference signals, making the angle error measurement independent of target range and radar cross-section.**

- **Difference (Δ) Pattern:**
  - **Shape: Features a sharp, deep null (zero point) precisely along the boresight direction. On either side of the null, the pattern has lobes with opposite phases.**
  - **Purpose: Provides the angle error information. The voltage output of the difference channel is very sensitive to small angular deviations from boresight. When the target is slightly off boresight, a significant difference signal is generated. The sign of this signal (relative to the sum signal) immediately indicates the direction of the error, and its magnitude (normalized by the sum signal) indicates the magnitude of the error.**

Application in Precise Angle Estimation:

The power of monopulse lies in its ability to extract angle information from a *single* received pulse. This offers several advantages for precise angle estimation:

1. **High Accuracy: The steep slope of the difference pattern around the null provides very high sensitivity to small angular deviations, leading to highly accurate angle measurements (often an order of magnitude better than the antenna beamwidth itself).**
2. **Immunity to Target Fluctuations: Since angle information is derived from a single pulse, rapid fluctuations in target amplitude (scintillation) do not introduce angle errors, unlike sequential lobing or conical scan systems that rely on amplitude comparisons between sequentially received pulses. This is a critical advantage for tracking maneuvering targets or those with fluctuating radar cross-sections.**
3. **Resistance to Jamming: Monopulse systems are generally more robust to certain types of jamming (e.g., amplitude-modulated noise jamming) because they compare signals simultaneously rather than sequentially.**
4. **Faster Update Rates: As only one pulse is needed for an angle estimate, monopulse radar can provide very fast angular updates, which is crucial for tracking agile targets.**
5. **Simultaneous Measurement: Both azimuth and elevation errors can be measured simultaneously from a single pulse.**

Numerical Example:

An X-band monopulse radar has an antenna with a beamwidth of 1.5 degrees. For its monopulse angle measurement, it can typically achieve angle tracking accuracy (RMS error) that is a fraction of its beamwidth, often 1/10 to 1/100 of the beamwidth.

Let's assume this radar can achieve an RMS angle accuracy of 1/50 of its beamwidth.

Given:

Beamwidth = 1.5 degrees

Fractional accuracy = 1/50

Angle Accuracy = Beamwidth / 50=1.5 degrees/50=0.03 degrees

This means that even with a 1.5-degree beamwidth, the monopulse technique allows the radar to determine the target's angular position with an accuracy of 0.03 degrees. This significantly exceeds the radar's nominal angular resolution and is why monopulse is essential for precise fire control, missile guidance, and high-accuracy tracking applications.

4.4 Advanced Tracking Algorithms

While the principles of target tracking involve prediction, association, and update, the actual mathematical implementation of these steps relies on sophisticated algorithms. These algorithms provide optimal estimation of target states by accounting for measurement noise and target motion dynamics.

4.4.1 Brief Overview of Kalman Filters and Other Common Tracking Algorithms

The Kalman Filter is arguably the most widely used and fundamental algorithm for state estimation in dynamic systems, including radar target tracking. It is an optimal recursive data processing algorithm that minimizes the mean square error of the estimated state, assuming linear system dynamics and Gaussian noise.

Kalman Filter Principles:

The Kalman filter operates in a two-step recursive process:

1. Prediction (Time Update): Based on the previous best state estimate, the filter predicts the current state of the target using a mathematical model of target motion (e.g., constant velocity, constant acceleration). It also predicts the uncertainty (covariance) of this predicted state.
    - Equation for predicted state: $\hat{x}_k^- = F_k \hat{x}_{k-1}^+ + B_k u_k$
    - Equation for predicted covariance: $P_k^- = F_k P_{k-1}^+ F_k^T + Q_k$
      where $\hat{x}$ is the state vector, F is the state transition matrix, B is the control input matrix, u is the control vector, P is the covariance matrix, and Q is the process noise covariance matrix.

The superscripts '−' and '+' denote 'before measurement update' and 'after measurement update', respectively.

2. Update (Measurement Update): When a new radar measurement arrives, the filter compares this measurement to its prediction. It then calculates a "Kalman Gain" that determines how much weight to give to the new measurement versus the prediction. Using this gain, it updates the state estimate, reducing its uncertainty.
    - Equation for Kalman Gain: $K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}$
    - Equation for updated state: $\hat{x}_k^+ = \hat{x}_k^- + K_k(z_k - H_k \hat{x}_k^-)$
    - Equation for updated covariance: $P_k^+ = (I - K_k H_k) P_k^-$
      where z_k is the measurement vector, H_k is the measurement matrix, and R_k is the measurement noise covariance matrix.

Advantages of Kalman Filter:

- Optimal for Linear Systems with Gaussian Noise: Provides the best possible estimate under these assumptions.
- Recursive: Only needs the previous state estimate and the current measurement, making it computationally efficient.
- Provides Uncertainty (Covariance): The filter inherently provides a measure of the uncertainty in its state estimates, which is crucial for track management (e.g., sizing track gates).

Limitations of Kalman Filter:

- Linearity Assumption: The standard Kalman filter assumes linear system dynamics and linear measurement models. Real-world target motion and radar measurements are often non-linear.

Other Common Tracking Algorithms (and extensions of Kalman Filter):

1. Extended Kalman Filter (EKF):
    - Principle: An extension of the Kalman filter for non-linear systems. It linearizes the non-linear system and measurement models around the current state estimate using Jacobian matrices.
    - Application: Widely used in radar tracking, as radar measurements (range, azimuth, elevation) are non-linear transformations of Cartesian position.
    - Limitation: Performance can degrade with highly non-linear dynamics or large estimation errors due to the linearization approximation.
2. Unscented Kalman Filter (UKF):

- **Principle: Another non-linear extension that addresses EKF's limitations. Instead of linearizing, it uses a deterministic sampling technique (sigma points) to propagate the mean and covariance through the non-linear transformations. This generally provides a more accurate approximation of the non-linear transformation.**
- **Advantage: Often more accurate and robust than EKF for highly non-linear systems.**

3. **Particle Filter (PF):**
   - **Principle: A more general non-linear, non-Gaussian filter. It represents the probability distribution of the target's state using a set of weighted "particles" (hypothesized states). These particles are propagated through the system dynamics and weighted based on how well they match new measurements.**
   - **Advantage: Can handle highly non-linear dynamics and non-Gaussian noise.**
   - **Limitation: Computationally much more intensive than Kalman-based filters, especially for high-dimensional state vectors.**
4. **Alpha-Beta (α-β) Filter:**
   - **Principle: A simpler, non-optimal, but computationally very inexpensive recursive filter often used for its simplicity in basic tracking applications. It uses two constant gain parameters, alpha (α) and beta (β), to smooth the current position and velocity estimates.**
   - **Advantages: Simple to implement, low computational cost.**
   - **Disadvantages: Not optimal; its performance is sensitive to the choice of α and β and does not adapt to changes in target dynamics or noise levels. No inherent estimate of covariance.**
5. **Interacting Multiple Model (IMM) Filter:**
   - **Principle: A powerful technique for tracking maneuvering targets. It runs multiple Kalman (or EKF/UKF) filters in parallel, each representing a different target motion model (e.g., constant velocity, constant acceleration, coordinated turn). It then probabilistically weights the outputs of these individual filters based on how well each model matches the observed measurements.**
   - **Advantages: Excellent performance for maneuvering targets, as it can dynamically switch between or blend motion models.**
   - **Application: Widely used in modern air traffic control and military tracking systems.**

In-depth Explanation:

The choice of tracking algorithm depends on the specific application's requirements, computational resources, and the expected target dynamics. For robust multi-target tracking in complex scenarios, algorithms like the IMM filter are often combined with data association techniques (e.g., Nearest Neighbor, Probabilistic Data Association Filter - PDAF, Multiple Hypothesis Tracking - MHT) to handle ambiguous measurements and maintain track integrity. The field of target tracking is an active area of research, continually evolving with advances in sensor technology and computational power.